EXPERIMENTS IN SCATTERED LIGHT. ANGLES OF FIRST MINIMUM

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ABSTRACT. Starting with Mie's formula, the distribution of intensity of light scattered in the region of the transmitted direction has been studied theoretically for four drop-sizes of water vapour, *viz.*, $\alpha = 15$, 20, 25 and 30, where α represents the ratio of the circumference of the drop to the wavelength of the light used. The range verified in the present investigations was 150°-180°. The calculations reveal that a sharp minimum of intensity exists in the range considered and that it has to be ascertained very carefully. This minimum, which is here termed as 'the angle of the first minimum on Mie's theory' may be correlated with the first order corona ring for the drop-sizes considered.

The theoretical results have been verified experimentally for the first time. The scattered intensity has been measured by the use of a photoelectric cell coupled with a suitable amplification arrangement. The results have been compared with Mie's theory and also with the first order corona rings. A fair agreement is found to exist in the cases considered.

Of all the methods so far used for measuring the instantaneous size of fog droplets, the corona ring method is very convenient. By applying Babinet's principle the formula

$$\sin \theta = (n + 0.22) \frac{\lambda}{2a}$$

is obtained, giving the relationship between the average radius of the drops in the cloud, the wavelength of light and the angle subtended by the *n*th dark ring.

The theory has been studied by C. T. R. Wilson (1897), Aitken (1890-91) and (1892-93), Bricard (1938), Rayleigh (1911) and several others, who found that it gives satisfactory results. While Mecke (1920) asserts that the theory fails to give reliable results for drops smaller than 4μ. Mitra (1928) confirms Mecke's view and reveals the oscillatory nature of diffraction pattern by means of photographs in monochromatic light. Recently T. A. S. Balkrishnan (1941) has put forth, following Mecke's and Mitra's views, a correction to the theory of coronas on the assumption that the droplets in the cloud are transparent. Paranjpe and Lajami (1939), have theoretically compared the angles of the first order corona rings for various drop-sizes with the angles of first minimum in the transmitted direction on Mie's (1908) theory.

Theoretical calculations for scattering by the application of Mie's theory at very close intervals for any drop-size in the transmitted direction, near about

180°, have revealed the presence of a sharp minimum of intensity in the scattered light. This minimum differs for different values of α where

$$\alpha = \frac{\text{Circumference of the drop}}{\text{Wavelength of light}} .$$

This minimum of intensity is here termed 'the angle of the first minimum on Mie's theory.'

Detailed derivation of Mie's formula is treated in a paper by Paranjpe and Lajami (1940). It is only necessary to indicate here the final results.

$$J_1 = \frac{\lambda^2}{4\pi^2 r^2} \left| \sum_{n=1}^{\infty} \left\{ A_n \pi_n + P_n \left[\nu \pi_n - (1-\nu^2)\pi_n^1 \right] \right\} \right|^2$$

$$J_2 = \frac{\lambda^2}{4\pi^2 r^2} \left| \sum_{n=1}^{\infty} \left\{ A_n \left[\nu \pi_n - (1-\nu^2)\pi_n^1 \right] + P_n \pi_n \right\} \right|^2$$

where J_1 and J_2 are intensity components of scattered light polarised in horizontal and vertical planes, λ the wavelength of incident light, and r the distance of point of observation from the centre of the scattering particle. The factor $\frac{\lambda^2}{4\pi^2 r^2}$ being constant the values of J_1 and J_2 depend on $|\text{Mod}|^2$ which may be termed as I_1 and I_2. A_n and P_n are complex functions depending on (*i*) the radius of the particle (*ii*) the wavelength of light used for scattering and

$$(iii) \quad m' = \frac{m}{\cdot}, \; i.e., \; \frac{\text{R. I. of the material of the particle}}{\text{R. I. of the surrounding medium}} .$$

A_n and P_n are calculated by evaluating series of coefficients in terms of cylindrical functions depending on α and β where $\alpha = \frac{2\pi\rho}{\lambda}$ and $\beta = m'\alpha$. π_n and π_n^1 are functions of the scattering angle θ derived from $\nu = \cos\theta$.

In Tables I and II below are given results of calculations of angles of first minimum on Mie's theory for scattering in the transmitted direction $(\pi - \nu°)$ for two drop-sizes $\alpha = 15$ and $\alpha = 16.8$.

TABLE I

$\alpha = 15$, $\beta = 20$

$\pi - \nu°$	I_1	I_2	$I = I_1 + I_2$
0°	23027.8	23027.8	46055.6
7° 15′	9502.3	8507.3	18009.6
10°	2466.3	2325.9	4792.2
12° 30′	1279.7	904.3	2184.0
15° 51′	98.5	128.8	227.3
17° 30′	33.0	90.6	123.6
20°	242.1	164.8	406.9
22° 45′	539.1	278.3	817.4
25° 23′	562.9	288.1	851.0
27° 3′	261.5	754.6	1016.1

TABLE II

$\alpha = 16.8$, $\beta = 22.4$

$\pi - \nu°$	I_1	I_2	$I = I_1 + I_2$
0	64140.5	64140.5	128281.0
7° 15	13189.3	9151.6	22340.9
10°	1523.7	1429.4	2953.1
12° 30′	359.6	342.3	701.9
15° 51′	982.6	908.5	1891.1
17° 30′	1343.1	1074.4	2417.5
20°	1074.4	994.8	2069.2

These values are plotted in Fig. 1. We see from the figure that the angles of the first minimum for the cases considered are :

Drop sizes		$\pi - r$
α 15,	β 20	16° 30′
α 16.8	β 22.4	12° 15′

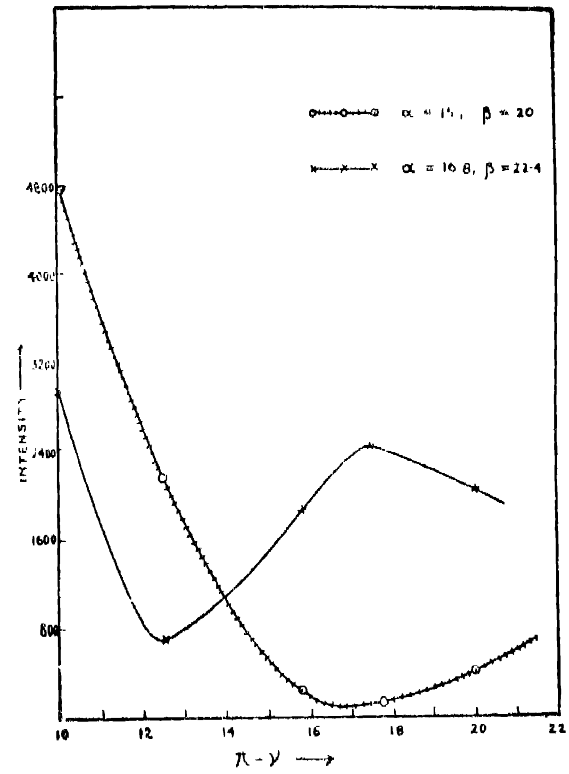

Fig. 1

It is of importance to observe that these angles $\pi - r$ are really the angles of first minimum as measured from the transmitted direction $(\pi - 0)$, and that there exists no other minimum in that interval. If sufficiently small intervals are not taken both in the theoretical and experimental work it is likely that this first order minimum may be missed altogether and the higher order minima might be mistaken for the first order one.

Ray (1921) calculated intensities for $\alpha = 12$, $\beta = 16$ on Rayleigh's theory and found out the angle for the first minimum near about $\pi - 27°30'$. While Paranjpe and Lajami (1930) calculating intensities at smaller intervals for the same case according to Mie's theory found out the angle of the first minimum near about $\pi - 14°$ and the angle of the second minimum near about $\pi - 27°30'$ Considering the above results it appears that Ray's value, namely $\pi - 27°30'$, should be in reality the value of second minimum on Rayleigh's theory, particularly since Mie's theory and Rayleigh's theory do not differ so widely from each other (*vide* Note by M. M. Paranjpe, 1942). If Ray had calculated intensities for angles between $\pi - 0°$ and $\pi - 27°30'$ he would have probably found a minimum between $\pi - 0°$ and $\pi - 27°30'$ and that too at about $\pi - 14°$ since the angles of second minimum by the two theories appear to be the same.

The following method may be employed to find out theoretically the angles of the first minimum. In Table III, I_1 and I_2 are given separately. I_1 and I_2 each consists of a real component and an imaginary component, which when squared and added together give I_1 and I_2.

TABLE III

$\alpha = 15,\ \beta = 20$

Angle $\pi - r$	I_1 Real	I_1 Imaginary	I_2 Real	I_2 Imaginary
0° 0′	+ 3.0	− 151.7	− 3.0	+ 151.7
7° 15′	+ 13.9	− 90.7	− 4.5	+ 92.6
10° 0′	+ 2.3	− 40.6	− 1.6	+ 48.2
12° 30′	+ 13.3	− 33.2	− 1.7	+ 52.2
15° 51′	+ 2.4	0.6	3.6	+ 10.8
17° 30′	− 4.8	3.1	+ 6.8	+ 6.7
20° 0′	− 15.5	+ 1.1	+ 11.7	+ 5.2
22° 45′	− 23.2	+ 1.5	+ 15.8	+ 5.5
25° 23′	− 23.7	+ 1.[illegible]	+ 16.0	+ 3.6
27° 3′	− 3.4	+ 15.8	+ 13.5	− 23.9

A careful examination of the above table reveals the following:

For I_1

(1) The real terms are smaller than the imaginary ones up to $\pi - 17°30'$. The squares of the real terms would be therefore smaller than those of the imaginary ones, and thus it can be seen that the values of I_1 in the interval $\pi - 0°$ and $\pi - 17°30'$ depend mainly on the values of the imaginary terms.

(2) The imaginary terms of I_1 decrease gradually in magnitude from -151.7 at $\pi - 0°$ to -3.1 at $\pi - 17°30'$. At $\pi - 20°$ the (imaginary) value is $+1.1$. It is clear from this that the imaginary values pass through zero and then change the sign at some point in the interval $\pi - 17°30'$ and $\pi - 20°$.

(3) From $\pi - 17°30'$ the real quantities for I_1 are greater than the imaginary ones. This state continues thereafter, indicating that I_1 has passed through a change in the values.

For I_2

(1) The real terms in I_2 are also small as compared to the imaginary ones till about $\pi - 15°51'$. The total value of I_2 can be supposed to be due, mainly to the value of the imaginary quantities in the interval $\pi - 0°$ to $\pi - 15°51'$.

(2) The imaginary terms in I_2 decrease continuously from $+151.7$ at $\pi - 0°$ to $+3.6$ at $\pi - 25°23'$. They pass through zero and change in sign from $+3.6$ to -23.9 somewhere in the interval between $\pi - 25°23'$ and $\pi - 27°3'$.

(3) But it is seen that the preponderence of imaginary values over the real ones ceases at $\pi - 17°30'$, where both the values are almost equal, *viz.*, $+6.8$ and $+6.7$. From this point onwards the real terms are mainly responsible for the values of I_2.

These inferences show clearly that the values of I_1 and I_2 decrease gradually in the interval $\pi - 0°$ to $\pi - 17°30'$.

Somewhere near about $\pi - 17°31'$ they pass through a minimum. Since the total intensity is equal to $I_1 + I_2$ it is clear that there is no minimum in the range $\pi - 0°$ to $\pi - 17°30'$, except the one investigated presently.

Other cases can be worked out similarly.

II

Experimental determination of the intensity of scattered light in the forward direction was made for various sizes of the drop using monochromatic sodium light. Scattered light was made incident on a photoelectric cell which could be moved freely to any desired angle. A new photoelectric amplification circuit was specially designed and constructed for measuring very feeble intensities in the scattered light. The amplified current was taken proportional to the intensity of scattered light. Readings of scattering were taken at intervals of about 2°30′ round the cloud chamber by receiving the scattered light on the photocell placed at the desired angles for the clouds of required particle size.

Great difficulty was experienced in finding the position of the minimum of intensity in the transmitted direction. The scattered light in this region is superposed on the diffraction rings and also mixed with direct light and the light diffused and reflected by the glass of the chamber. The incident beam was therefore cut down to a very narrow pencil which reduced the chance of getting pronounced corona rings and also kept out the direct and the diffused light from falling on the photocell.

Results of scattering in the forward direction are given in Table IV and their curves drawn in Fig. 2. A reference to the figure shows that the angles of first minimum are not so well defined as in the cases of other minima. In fact the minimum is only to be suspected in some of these cases. The following procedure is adopted: When intensity values for consecutive scattering angles are very near to each other, that is, the ratio between these consecutive readings is much less than those of the preceding and succeeding pairs, a minimum is suspected somewhere near about that angle. The reason for lack of sharpness in the nature of these minima is that there is a vast amount of scattered light in this region, since forward scattering is much more prominent than in other regions. There is besides scattered light, a large amount of extra light in this region due to reflections and refractions of the incident beam of light by the glass of the cloud chamber.

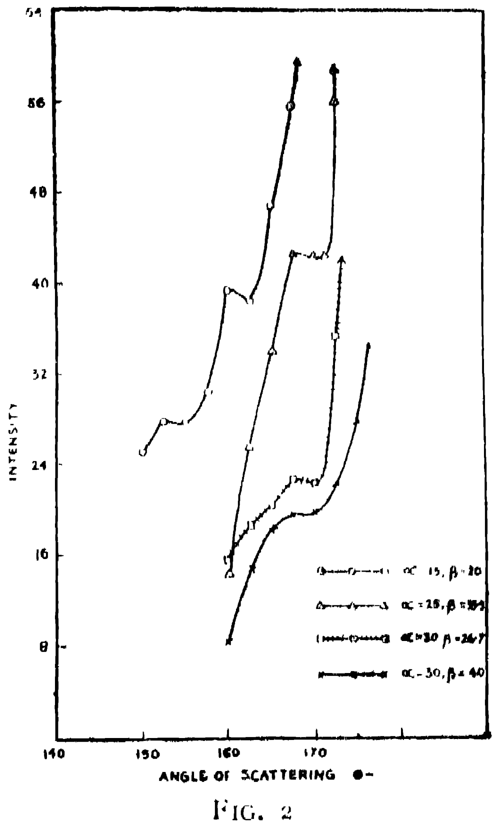

FIG. 2

In Fig. : the curves are drawn both for theoretical and experimental values of the drops $\alpha = 15$, 20, 25 and 30.

The curves are seen to show a fair agreement between theory and experiment.

Below is given a table showing a comparison between the theoretical and experimental data.

TABLE IV

(*a*)

$\alpha = 15$, $\beta = 20$.

Averaged over a set of 9 independent observations.

Scattering angle	Theoretical scattering	Observed scattering.
150°	1990.1	25.0
152° 57′	1016.2	27.6
154° 37′	851.0	27.6
157° 15′	817.4	30.4
160°	406.0	39.4
162° 30′	123.6	38.2
164° 9′	227.3	40.8
167° 30′	2184.0	55.6
170°	4797.2	.
172° 45′	18099.6	Readings beyond scale
180°	46055.6	...

(*b*)

$\alpha = 20$, $\beta = 26.7$.

Averaged over a set of 4 independent observations.

Scattering angle.	Theoretical scattering.	Observed scattering
160°	1370.1	15.7
162° 30′	...	18.5
165°	...	20.1
167° 30′	...	22.6
170°	2422.0	22.1
171° 53	1481.0	...
172° 30	3327.0	35.2
175°	...	...
177° 30′	...	...
180°	91742.2	134.3

(*c*)

$\alpha = 25$, $\beta = 33.33$.

Averaged over a set of 4 independent observations.

Scattering angle.	Theoretical scattering.	Observed scattering.
160°	Theoretical values not available.	14.3
162° 30		25.5
165°		34.0
167° 30′	...	42.7
170°	...	42.5
171° 15′	...	42.4
172° 30′	...	56.0
175°	...	...
177° 30′	...	Readings beyond scale.
180°	...	

(*d*)

$\alpha = 30$, $\beta = 40$.

Averaged over a set of 4 independent observations.

Scattering angle	Theoretical scattering	Observed scattering.
160°	...	16.3
162° 30	...	29.2
165°	...	36.4
167° 30′		39.5
170°	19121.2	39.6
171° 53	22828.0	..
172° 45′	12297.0	44.5
175°	...	55.3
177° 30′	...	Readings beyond scale.
180°	391438.0	

(1) Angles of first minimum obtained on Circular Disc theory $\pi - \nu_c$, (2) angles of first minimum calculated on Mie's theory $\pi - \nu_m$ and (3) angles of first minimum obtained experimentally $\pi - \nu_e$

TABLE V

Drop-size		$\pi-\nu_c$	$\pi-\nu_m$	$\pi-\nu_e$
$\alpha=30$	$\beta=40$	7° 20'	6° 30'*	11°
$\alpha=25$	$\beta=33.3$	8° 49	...	9°
$\alpha=20$	$\beta=26.07$	11° 0	9° 0'*	10°
$\alpha=16.8$	$\beta=22.4$	13° 10	12° 15'	...
$\alpha=15$	$\beta=20$	14° 48'	10° 36'	17° 30'
$\alpha=12$	$\beta=16$	18° 30	14° 0'	...
$\alpha=10$	$\beta=13.3$	22° 30	16° 0'*	...
$\alpha=9$	$\beta=12$	25° 12	18° 0'*	...

(Results marked with asterisk are taken from calculations by Paranjpe and Lajami.)

The above values are plotted against α in Fig. 3.

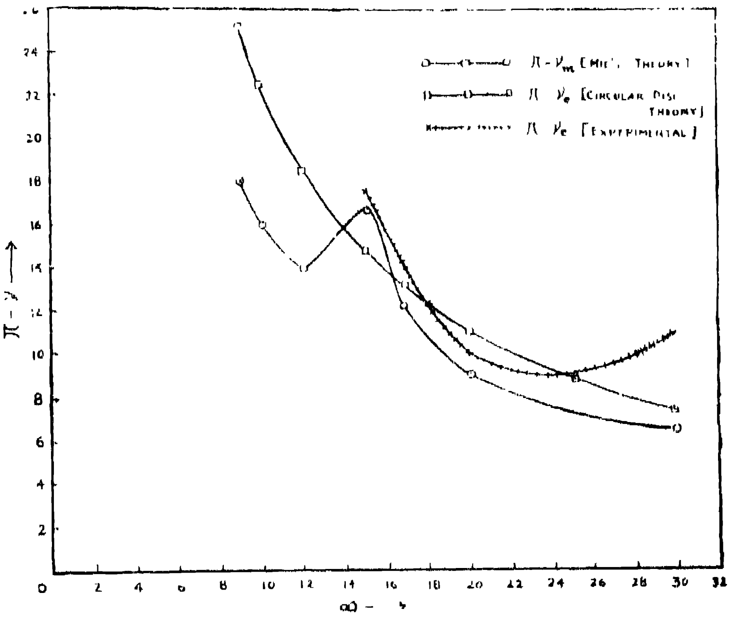

FIG. 3

The curve $\pi-\nu_c$ (Circular Disc theory) is a regular curve showing a sine relationship between α and the angle of the first minimum.

This curve $\pi-\nu_c$ shows that the value of the first minimum $\pi-\nu$ increases as the drop-size decreases. This result is perfectly in accordance with the Circular Disc Theory.

The curve $\pi-\nu_m$ (Mie's theory) exhibits the presence of a well defined maximum at $\alpha=15$ and a defined minimum at $\alpha=12$. These reversing points cannot be accounted for but it appears probable that the curve $\pi-\nu_m$ may exhibit more maxima if a sufficient number of points are plotted. It is seen from the figure that $\pi-\nu_m$ crosses $\pi-\nu_c$ at $\alpha=16.4$ and at $\alpha=14$, the two curves draw closer to each other at $\alpha=30$ and may cross at $\alpha=33$, if both of them were produced beyond 30. It thus appears that the curve $\pi-\nu_m$ is of an oscillatory nature interlinking itself with the curve $\pi-\nu_c$ by crossing it a number of times. No definite conclusion can be drawn from this behaviour because of the meagre data available. If an oscillatory nature is to be assumed for $\pi - \nu_m$ and its interlinking with $\pi - \nu_c$, this might lend support to the nature of the correction to be applied to the theory of the coronas as derived by Balkrishnan (*loc. cit.*), who concludes that the correction to the Circular Disc theory is of a periodically oscillatory nature.

The curve $\pi - \nu_e$ (Experimental) drawn with the points available appears to be of the same nature as $\pi - \nu_m$. It is seen that both these curves are almost parallel except at $a = 30$ where the experimental value is much higher than that expected theoretically. In other respects the curve appears to be identical with $\pi - \nu_m$ except that it is laterally shifted to higher values of $\pi - \nu$. It was not possible to extend the experimental verification below $a = 15$ because of the experimental limitations. If it were possible a complete relation between $\pi - \nu_m$, $\pi - \nu_e$ and $\pi - \nu_c$ could have been established.

Judging from the results available, it can be concluded that the theoretical and the experimental curves for angles of first minimum are almost identical in nature and it may be said that they agree considerably within the limits of the experimental errors. The experimental values appear to be slightly higher than the theoretical ones. But, in general, deviation from the theory is such, as would be accounted for by considering the difficulty in spotting out the minimum experimentally.

It is thus possible to employ Mie's theory as a method of determining the particle size of large water droplets. The method is found to be capable of giving reliable results in the range 1μ to 4μ.

The above discussion leads to another important conclusion, namely, that the Circular Disc theory is certainly not as defective as is thought by Meeke (*loc. cit.*) who concludes that the theory fails to give correct results below 4μ. In the present investigations drop-sizes considered were between 1μ to 4μ and the results are found to agree with those on Mie's theory, and the method can be used as a fair estimate of the size of droplets considered.

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